Exercise 7H: Solutions

1 a $2 : 4 : 6 : 8 = 1 : 2 : 3 : 4$

b $2 : 8 : 18 : 32 = 1 : 4 : 9 : 16$

c The second ratio is the square of the first.

2 a $2 : 4 : 6 : 8 = 1 : 2 : 3 : 4$

b $1 : 4 : 9 : 16$

c The second ratio is the square of the first.

3
$$\frac{A'B'}{AB} = \frac{5}{3}$$
$$\text{Area } A'B'C'D' = 7 \times \left(\frac{5}{3}\right)^2$$
$$= \frac{7 \times 25}{9}$$
$$= 19\frac{4}{9} \text{ cm}^2$$

4
$$\frac{20}{2.1^2} = \frac{20}{4.41}$$
$$= 4.54 \text{ cm}^2$$

5 a F is the midpoint of AC, so $AF = 1$ cm.
$$BF^2 = BA^2 - AF^2$$
$$= 2^2 - 1^2 = 3$$
$$BF = \sqrt{3} \text{ cm}$$

b
$$\frac{A'C'}{AC} = \frac{B'F'}{BF}$$
$$\frac{a}{2} = \frac{2}{\sqrt{3}}$$
$$a = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

c
$$\frac{\text{Area } A'B'C'}{\text{Area } ABC} = \left(\frac{B'F'}{BF}\right)^2$$
$$= \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3}$$

6
$$\text{Area ratio} = 16 : 25$$
$$\text{Side ratio} = \sqrt{\frac{16}{25}}$$
$$= \sqrt{\frac{4^2}{5^2}} = 4 : 5$$

7
$$30 \times \frac{9}{12} = 22.5 \text{ cm}$$

8 a $1 : 2 : 3$

b $1 : 2 : 3$

c $1 : 8 : 27$

d The third ratio is the cube of the first.

9 a i $8 : 12 = 2 : 3$

ii $4 : 6 = 2 : 3$

iii $3 : 4\frac{1}{2} = 2 : 3$

b $8 \times 4 \times 3 : 12 \times 6 \times 4\frac{1}{2} = 96 : 324$

$= 8 : 27$

c The ratio in **b** is the cube of the ratios in **a**.

10a $3 : 2 : 5$

b Sphere 1: $V = \frac{4}{3} \times \pi \times 3^3 = 36\pi$

Sphere 2: $V = \frac{4}{3} \times \pi \times 2^3 = \frac{32\pi}{3}$

Sphere 3: $V = \frac{4}{3} \times \pi \times 5^3 = \frac{500\pi}{3}$

$36 : \frac{32}{3} : \frac{500}{3} = 108 : 32 : 500$

$= 27 : 8 : 125$

c The second ratio is the cube of the first.

11 $(2 : 1)^3 = 2^3 : 1^3$

$= 8 : 1$

12 $(3 : 4)^3 = 3^3 : 4^3$

$= 27 : 64$

13 $\sqrt[3]{8 : 27} = \sqrt[3]{8} : \sqrt[3]{27}$

$= 2 : 3$

14 $\text{Volume ratio} = 64 : 27$

a $\text{Height ratio} = \sqrt[3]{64 : 27}$

$= 4 : 3$

b $\text{Radius ratio} = \sqrt[3]{64 : 27}$

$= 4 : 3$

15 $\text{Height ratio} = 2 : 1$

a $\text{Area ratio} = (2 : 1)^2$

$= 4 : 1$

b $\text{Capacity ratio} = (2 : 1)^3$

$= 8 : 1$

16a $(1 : 10)^2 = 1 : 100$

b $(1 : 10)^3 = 1 : 1000$

c $(1 : 10)^1 = 1 : 10$

d Both models will have the same number of wheels, so $1 : 1$.

17 $\frac{1}{2} \times \left(\frac{12}{8}\right)^3 = \frac{1}{2} \times \left(\frac{3}{2}\right)^3$

$= \frac{27}{16}$ litres

$\frac{1}{2} \times \left(\frac{16}{8}\right)^3 = \frac{1}{2} \times 2^3$

$= 4$ litres

18 $343 \times \left(\frac{7.5}{10.5}\right)^3 = 343 \times \left(\frac{5}{7}\right)^3$

$= 125$ mL

$343 \times \left(\frac{9}{10.5}\right)^3 = 343 \times \left(\frac{6}{7}\right)^3$

$= 216$ mL

19a Length ratio $= \sqrt{1 : 2500}$

$= 1 : 50$

b Capacity ratio $=$ (area ratio)^3

$= (1 : 50)^3$

$= 1 : 125\,000$

c Width $= 150 \times \frac{1}{50}$

$= 3$ cm

d Area $= 3 \div \frac{1}{2500}$

$= 3 \times 2500 = 7500$ cm^2

20a Height ratio $= \sqrt{144 : 169}$

$= 12 : 13$

b Capacity ratio $= (12 : 13)^3$

$= 1728 : 2197$

21a Ratio of sides $= 1 : 2$

Ratio of areas $= 1^2 : 2^2 = 1 : 4$

Four times

b Area $\triangle AKM = \frac{15}{4} = 3.75$

22 $\triangle BDE \sim \triangle CAF$

and $AB = AC = 2AD$

$\therefore \quad BD^2 = BA^2 - AD^2$

$= (2AD)^2 - AD^2$

$= 3AD^2$

Ratio of areas $= \frac{BD^2}{AC^2}$

$= \frac{3AD^2}{(2AD)^2}$

$= \frac{3AD^2}{4AD^2} = \frac{3}{4}$

So the ratio is $3{:}4$

Note: It is easier to express lengths in terms of AD as fractions are avoided.

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$$\begin{aligned}\text{Area ratio} &= 144 : 81 \\ &= 12^2 : 9^2\end{aligned}$$

$$\text{Length ratio} = 12 : 9$$

$$\begin{aligned}\text{Length in second triangle} &= \frac{9}{12} \times 6 \\ &= 4.5 \text{ cm}\end{aligned}$$